THERMOPHYSICAL ANALYSIS OF 25 %, 37 %, and 45 % AQUEOUS PROPYLENE GLYCOL SOLUTIONS FOR HELIOSYSTEMS AND THEIR ANALYTICAL CALCULATION

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Abstract. The paper analyzes the thermophysical properties of 25 %, 37 %, and 45 % aqueous solutions of propylene glycol for heliosystems as heat carriers. A computer simulation of the movement of these coolants in the pipe space of the heliosystem, with a constant velocity $V = 0.93$ m/s, was carried out. Using classic numerical empirical equations, the thermophysical and hydrodynamic characteristics of these glycol solutions in the temperature range of 243–373 K were found. The distribution of velocity vectors in the "live section" of the tubular space of the solar system follows a quadratic parabola, while the distribution of turbulent heat conductivity and, accordingly, temperatures follows a cubic parabola. A cubic numerical equation, the real root of which is the dimensionless number Bl_{turb}, was analytically derived for determining heat transfer coefficients of coolants at any temperature and velocity in the turbulent regime. The distribution of turbulent thermal conductivities k_{turb} (W/m·K) (as well as temperatures, T (K) and velocities V (m/s) in a flow with free turbulence for an aqueous solution of 37 % propylene glycol at an axial velocity in the center of the flow core $V = 0.93$ m/s is shown graphically at a temperature of 343 K in the pipe space of the heliosystem with a diameter $D = 0.021$ m. A relation for finding Bl_{turb} numbers for transient modes of motion is obtained, which is mainly implemented in heliosystems.

Keywords: transient, turbulent viscosity, thermal conductivity, average thickness of the laminar boundary layer (LBL), surface tension coefficient of the coolant, analytical cubic numerical equation.

1. Introduction

In the modern realities of the global world political and economic crisis, renewable energy sources come to the fore in the energy sector of Ukraine. Actually, thermal generation, which is based mainly on fossil fuel – coal – is completely destroyed; that is, thermal power plants are essentially beyond repair, and their restoration is not advisable for a number of reasons. First of all, such thermal power plants are very vulnerable to the future, since the energy capacities are concentrated in a relatively small area. Secondly, this type of energy is already completely out of date, as it creates significant environmental problems. Thirdly, it is very energy-consuming and in market conditions implies high electricity prices for both industrial and household consumers.

In the current conditions, liquid-phase heliosystems (solar collectors) that do not require large energy costs for their production and do not carry any environmental problems, such as, for example, solar panels, which require significant capital investments for their disposal, are almost the first place when applied for about 50 years, *i. e.*, in the future. In addition, such energy objects are scattered and cannot be destroyed at once. In addition, solar collectors have much higher efficiency (about 90 %), while, for example, solar panels' efficiency is less than 25 %.

The operation of solar collectors has a number of features. It is advisable to operate such solar systems in winter at sub-zero temperatures, when the water crystallizes and needs to be replaced. For this reason, in solar systems, aqueous solutions of glycols are used (ethylene glycol, propylene glycol, *etc.*), which

significantly lower the crystallization temperature. At the same time, the addition of glycols significantly increases the dynamic viscosity coefficient and, as a result, reduces the coolants' velocity and, accordingly, the heat transfer coefficient. For this reason, aqueous solutions of glycols (ethylene glycol, propylene glycol, *etc.*) are used in solar systems. In food technologies, it is advisable to use aqueous solutions of propylene glycol, since ethylene glycol is somewhat toxic and its contact with food products is not permissible. The most common are aqueous solutions of propylene glycol with concentrations of 25 %, 37 %, and 45 %, the crystallization temperatures of which are 263 K, 253 K, and 243 K, respectively.

In order to compensate for the shortcomings of glycols, researchers currently use them in combination with various nanofluids, which significantly improve the thermophysical properties of both water and aqueous solutions of glycol. Moreover, the effect of nanofluids as an additive to the base fluid is more favorable for glycols than for water (Fig. 1).[1]
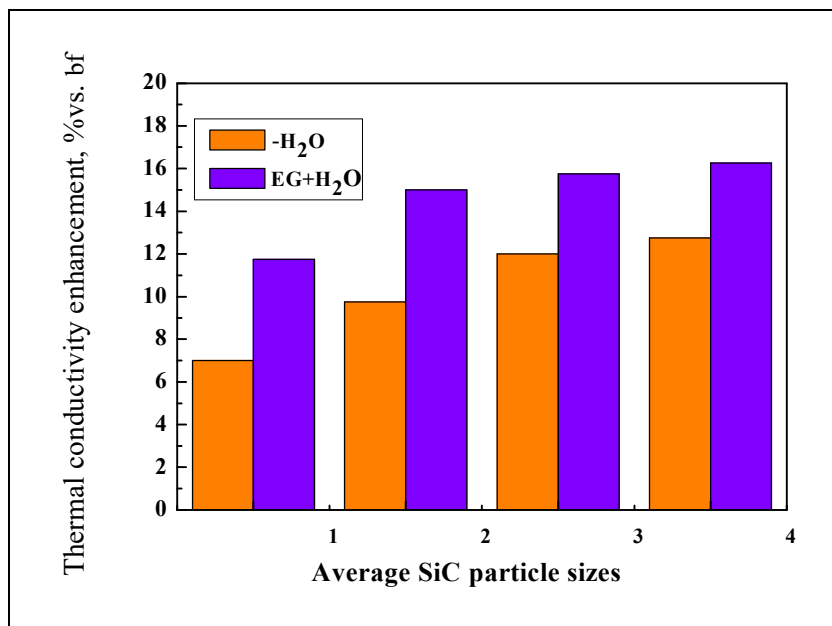

Fig. 1. Comparison of the increase in thermal conductivity in 4 vol. % SiC nanofluids
with EG/H$_2$O (50:50) and H$_2$O as base fluids at different nanoparticles sizes:
1–16 nm; 2–28 nm; 3–66 nm; 4–90 nm. [1]It can be seen from (Fig. 1)
that the basic aqueous solution of ethylene glycol with the EG/H2O ratio of 50:50
and the addition of SiC nanofluid has different average diameters of nanoparticles
in the range of 16–90 nm and behaves approximately more efficiently (by 25–30 %) than water[1]

To optimize the qualitative and quantitative composition of nanofluid additives to coolants, most researchers on this issue proceed through empirical determination of the thermophysical quantities of nanofluids by generalizing many experimental data. These problems were investigated in our previous works.[2, 3] Such an empirical approach is also included in the change of numerical calculation equations for the use of nanofluids. At the same time, numerical equations acquire complex forms and are difficult to calculate (see Tables 1, 2).[2, 3] In addition, to use new nanoliquids, for example, to increase the efficiency of solar collectors, it is necessary to first conduct expensive and time-consuming experiments to generalize empirical data into criterion numerical equations, and only then it will be possible to theoretically calculate and design such solar systems.

We would like to raise another issue that we believe to be very important. Heat exchange equipment, which is widely used in various industries, including food, chemical, and pharmaceutical technologies, typically operates in turbulent (*T*) or transitional (*Tr*) modes of heat transfer fluid movement, as they are significantly more efficient due to their relatively high velocity. Therefore, the movement of coolants should be considered not in static terms, but in dynamic terms, *i. e.*, molecular (stationary) characteristics of coolants should not be used. Molecular characteristics describe the thermal motion of molecules in a liquid coolant, rather than the turbulent motion of flows involving a convective component, where the turbulent viscosity and thermal conductivity of heat agents are several orders of magnitude higher than molecular ones. For the first time, such an approach to the interpretation of turbulent viscosity

was given in his works by the French scientist J. Boussinesq, who proposed to consider turbulent fluid flow as "Newtonian" and suggested that the behavior of free fluid jets is the result of the combined action of surface and gravitational forces.

In solar collectors, propylene glycol solutions have a relatively high viscosity, which limits high flow rates. It is important that for the efficient operation of solar systems, it is necessary to ensure the maximum possible heat transfer coefficient. On the one hand, at the maximum possible velocities of coolants, we have high heat transfer coefficients. On the other hand, high velocities require large hydraulic resistances and are also limited by the high viscosity of coolants.

To accurately calculate the optimal operation of the solar system, it is necessary, in our opinion, to come across the boundary between laminar (*L*) and *Tr* modes, where we have a relatively low velocity, but there is a significant turbulence of the flow, which strengthens the convective component and increases the overall heat transfer coefficient. A detailed analysis of this *Tr* is made in article.[5] Article[5] presents the experimental dependence of the friction coefficient *f* on the Reynolds number in the range of $3000 < Re < 10000$, *i. e.*, within the limits of the existence *Tr* regime. We are interested in this range of Reynolds numbers for the use of aqueous solutions of propylene glycol of three concentrations. It should be noted that the coefficients *f*, which are responsible for the mechanical component of the friction force, were studied in article.[4] On the other hand, under the conditions of vibrational microdisplacements, an important role is played by the adhesive component of the frictional forces,[5] which in aviation engineering is determined taking into account the surface energy of the metal.[6] Such an approach is no less important for liquid coolants.[2, 3] It turned out that the classical numerical equations used for the calculation and design of liquid heat exchangers, in particular solar collectors, are insensitive to the characteristics of the surface of heat carriers, since they do not contain a series of similarity numbers with such characteristics. At the same time, such a calculation is inaccurate and does not reflect the real picture of heat exchange when using glycols, as well as with the addition of nanofluids.[2, 3.]

Based on the above, it is currently relevant to search for new analytical equations using turbulent values of coolant flows, taking into account surface characteristics for quick, convenient calculation and design of modern heliosystems, in particular with the use of glycols.

2. Experimental

The main idea of computer experiments (simulations) was to carry out all calculations at a constant

velocity ($V = 0.93$ m/s) of heat carrier in the pipe space of the heliosystem with a diameter $D = 0.021$ m to fix the transitions of motion modes ($L{\to}Tr$) and ($Tr{\to}T$) from the ratios between the main thermophysical characteristics of flows when their temperature changes. We chose the velocity $V = 0.93$ m/s for the following reasons. At this velocity, the given aqueous solutions of propylene glycol of the specified concentrations in the specified temperature range, *i. e.*, their existence in the liquid phase state, have two regime transitions *L–Tr* and *Tr–T*. As is known, the Reynolds number also depends on the velocity. A constant velocity is necessary to see other patterns. If we significantly change the velocity of the coolants, then in a solution with a minimum concentration of 25 % in this temperature range, the *L* regime disappears, and in a solution with a maximum concentration of 45 % the *T* regime disappears. For our computer modeling, it was important to see the change in the hydromechanical characteristics of the selected solutions in three regimes when their temperature changes. Tables 1, 2, and 3 show the main thermophysical characteristics of aqueous solutions of propylene glycol with concentrations of 25 %, 37 %, 45 %, which are widely used in liquid solar systems, depending on the temperature (columns 1–6). All further calculations of the relevant thermophysical and hydrodynamic characteristics were carried out in the system of computer simulation of coolant flows.

Columns 7, 8 show the Reynolds and Prandtl numbers with motion modes at a velocity of $V = 0.93$ m/s in the tube space of the heliosystem with a diameter of $D = 0.021$ m. The crystallization temperatures of such solutions are (263 K – 25 %), (253 K – 37 %), (243 K – 45 %).

Columns 9 (Tables 1–3) offer the friction coefficients for modes *L*, *Tr*, and *T*, calculated according to various known empirical equations for individual modes, respectively. For comparison, friction coefficients were calculated for *Tr* and *T* modes according to various known formulas (1) and (2) (Tables 1–3), numerator / denominator, respectively.

$$f_1 = 64/Re \quad (L)$$
$$f_1 = 0.316/Re^{0,25} \quad (T, Tr) \tag{1}$$
$$f_2 = 64/Re \ (L)$$
$$f_2 = (0.79\ln Re - 1{,}64)^{-2}(Tr, T) \tag{2}$$

Columns 10 present the heat transfer coefficients h, which were calculated according to classical well-known numerical equations for *L*, *Tr* and *T* modes, respectively (Formulas (3–6)), where (3), the known Gnelinsky formula for *Tr* and *T* modes of movement of heat carriers, and (4, 5, 6) are known empirical equations for *L*, *Tr* and *T* modes.

Table 1. Thermophysical properties of a 25 % aqueous solution of propylene glycol (crystallization temperature is 263 K, $V = 0.93$ m/s, $D = 0.021$ m)*

Temperature, T, K	Density, kg/m³	Heat capacity, C_p, J/kg·K	Thermal conductivity, k, W/m·K	Dynamic viscosity, $\mu \cdot 10^{-3}$, N·s/m²	Surface tension, $\sigma \cdot 10^{-3}$, N/m	Reynolds number. Regime Re	Prandtl number, Pr	f_1, $f_2 10^{-2}$ from eq. (1/2)	$h_{classic}$, W/m²K, from equations (3–6)	a, according to the ratio (7) using from eq. (3–5)	Bilonoga number, Bl from eq. (8)	Bilonoga number turb, Bl_{turb} from eq. (9)	k_{trans}, W/mK, from eq. (10)	k_{turb}, W/mK, from eq. (11)	δ_{LBL}-average thickness LBL 10^{-3}[m] from eq. (12)	h_{new} W/m²K, from eq.(15/16)	h_{new}, W/m²K, from eq.(25)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
263	**1032**	**3.93**	**0.466**	**10.22**	**58.8**	**1966 (L)**	**86.2**	**3.26** **3.26**	**313**	**0.00** **0.00**	**10.89**	**0.98**	**3.69**	**0.00**	**10.5**	**351**	**351.4**
273	1030	3.95	0.470	6.18	57.7	3245 (Tr)	51.9	4.19	1415 1124	0.0019 0.0012	6.73	0.63 1.05	3.63	2.27 3.80	6.45 5.12	1418 1125	1265
283	1027	3.96	0.474	4.52	56.6	4423 (Tr)	37.8	3.87	1646 1467	0.0031 0.0025	5.03	1.16 1.43	3.56	4.14 5.08	4.86 4.32	1648 1465	1469
293	1024	3.98	0.478	2.86	55.5	6970 (Tr)	23.8	3.46	2048 2029	0.0054 0.0053	3.25	2.04 2.07	3.50	7.15 7.25	3.45 3.42	2047 2029	1933
303	**1020**	**3.99**	**0.484**	**2.14**	**53.2**	**9279 (Tr)**	**17.6**	**3.22**	**2358** **2438**	**0.0078** **0.0081**	**2.54**	**2.81** **2.68**	**3.36**	**9.44** **9.02**	**2.76** **2.85**	**2358** **2435**	**2265**
313	1016	4.00	0.491	1.42	51.1	13929 (T)	11.6	2.91	2907 3087	0.0127 0.0137	1.76	4.02 3.72	3.23	12.98 12.03	2.09 2.22	2910 3091	2972
323	1009	4.02	0.498	1.16	48.9	16933 (T)	9.4	2.77	3147 3444	0.0156 0.0175	1.50	4.83 4.33	3.10	14.98 13.42	1.80 1.97	3147 3444	3318
333	1003	4.03	0.505	0.903	46.7	21623 (T)	7.2	2.61	3468 3911	0.0201 0.0232	1.23	5.93 5.15	2.96	17.57 15.24	1.52 1.71	3462 3895	3851
343	994	4.04	0.512	0.787	44.4	24587 (T)	6.2	2.52	3655 4176	0.0232 0.0271	1.13	6.77 5.80	2.82	19.10 16.37	1.35 1.54	3662 4178	4045
353	986	4.05	0.519	0.671	42.1	28606 (T)	5.2	2.43	3886 4494	0.0273 0.0322	1.01	7.80 6.61	2.68	20.91 17.72	1.19 1.38	3884 4504	4338
363	983	4.06	0.526	0.590	39.7	32434 (T)	4.6	2.35	4103 4780	0.0311 0.0369	0.95	7.51 8.92	2.53	19.01 22.57	1.06 1.23	4144 4774	4472
373	979	4.08	0.533	0.509	37.5	37442 (T)	3.9	2.27	4359 5107	0.0361 0.0430	0.87	10.17 8.54	2.40	24.42 20.49	0.94 1.10	4363 5106	4736

- The values in bold font indicate the boundaries of the *L–Tr* and *Tr–T* regime transitions. The critical values of the numbers *Bl* and *Bl_{turb}* are highlighted on a gray background

Table 2. Thermophysical properties of a 37 % aqueous solution of propylene glycol (crystallization temperature is 53 K; $V = 0.93$ m/s, $D = 0.021$ m)*

Temperature, T, K	Density, kg/m³	Heat capacity C_p, J/kg·K	Thermal conductivity, k, W/m·K	Dynamic viscosity $\mu \cdot 10^{-3}$, N·s/m²	Surface tension $\sigma \cdot 10^{-3}$, N/m	Reynolds number. Regime Re	Prandtl number, Pr	f_1, f_2 10^{-2} from eq. (1/2)	$h_{classic}$, W/m²K from equations(3–6)	a, according to the ratio (7) using from eq. (3–5)	Bilonoga number, Bl from eq. (8)	Bilonoga number turb, Bl_{turb} from eq. (9)	k_{trans}, W/m·K from eq. (10)	k_{turb}, W/m·K from eq. (11)	δ_{LBL} - average thickness LBL 10^{-3}, m, from eq. (12)	h_{new}, W/m²K from eq. (15/16)	h_{new}, W/m²K, from eq. (25)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
253	1050	3.68	0.420	47.25	54.29	434 (L)	414	14.91 / 14.8	313	0.00	52.8	0.00	3.29	0.00	10.5	313	**313.3**
263	1047	3.70	0.422	29.89	53.27	684 (L)	262	9.38 / 9.38	309	0.00	34.1	0.00	3.24	0.00	10.5	308	**308.6**
273	1045	3.72	0.425	12.54	52.25	1628 (L)	110	3.95 / 3.95	304	0.00	14.6	0.00	3.19	0.00	10.5	304	**303.8**
283	**1040**	**3.74**	**0.427**	**8.550**	**51.22**	**2375 (Tr)**	**74.89**	**4.53 / 4.94**	**1133 / 747**	**0.0013 / 0.0004**	**10.2**	**0.25 / 0.90**	**3.13**	**0.79 / 2.82**	**8.40 / 5.53**	**1181 / 745**	**1061**
293	1036	3.77	0.429	4.56	50.19	4437 (Tr)	40.07	3.87 / 4.01	1527 / 1354	0.0031 / 0.0025	5.58	2.88 / 1.60	3.08	4.03 / 4.93	2.71 / 4.04	1525 / 2273	**1531**
303	1033	3.79	0.431	3.410	47.98	5899 (Tr)	29.99	3.61 / 3.67	1750 / 1670	0.0044 / 0.0041	4.38	1.97 / 2.11	2.95	5.81 / 6.23	3.54 / 3.38	1746 / 1667	**1731**
313	**1022**	**3.82**	**0.433**	**2.26**	**45.77**	**8858 (Tr)**	**19.94**	**3.26 / 3.26**	**2127 / 2174**	**0.0075 / 0.0073**	**3.05**	**3.04 / 2.95**	**2.83**	**8.59 / 8.34**	**2.60 / 2.66**	**2128 / 2177**	**2183**
323	1024	3.85	0.435	1.79	43.55	11107 (T)	15.84	3.08 / 3.06	2453 / 2489	0.0099 / 0.0101	2.55	3.84 / 3.77	2.70	10.4 / 10.2	2.17 / 2.20	2455 / 2488	**2515**
333	1012	3.88	0.437	1.32	41.35	14973 (T)	11.72	2.86 / 2.82	2749 / 2941	0.0134 / 0.0145	1.99	4.98 / 4.40	2.58	12.9 / 11.4	1.76 / 2.00	2580 / 2932	**3027**
343	1007	3.91	0.439	1.11	39.19	17665 (T)	9.79	2.74 / 2.70	3211 / 2930	0.0159 / 0.0177	1.77	5.88 / 5.28	2.45	14.4 / 12.9	1.53 / 1.67	3207 / 2934	**3239**
353	997	3.94	0.441	0.897	36.95	21707 (T)	8.01	2.61 / 2.56	3566 / 3171	0.0199 / 0.0223	1.52	7.07 / 6.18	2.32	16.4 / 14.3	1.30 / 1.46	3569 / 3178	**3579**
363	989	3.97	0.443	0.792	34.84	24388 (T)	7.10	2.53 / 2.49	3779 / 3319	0.0222 / 0.0254	1.43	8.02 / 6.92	2.20	17.6 / 15.2	1.16 / 1.33	3793 / 3308	**3759**
373	982	4.00	0.445	0.687	32.60	27916 (T)	6.18	2.45 / 2.41	4032 / 3499	0.0252 / 0.0295	1.33	9.28 / 7.92	2.06	19.1 / 16.3	1.02 / 1.18	4039 / 3492	**3728**

* The values in bold font indicate the boundaries of the *L–Tr* and *Tr–T* regime transitions. The critical values of the numbers Bl and Bl_{turb} are highlighted on a gray background

Table 3. Thermophysical properties of a 45 % aqueous solution of propylene glycol (crystallization temperature is 243 K; V = 0.93 m/s, D = 0.021 m)*

Temperature, T, K	Density, kg/m³	Heat capacity C_p, J/kgK	Thermal conductivity, k, W/mK	Dynamicviscosity $\mu \cdot 10^{-3}$, N·s/m²	Surfacetension $\sigma \cdot 10^{-3}$, N/m	Reynolds number. Regime Re	Prandtl number, Pr	f_1, f_2 10^{-2} from eq. (1/2)	$h_{classic}$, W/m²K, from equations (3–6)	a, according to the ratio (7) using from eq. (3–5)	Bilonoga number, Bl from eq. (8)	Bilonoga number turb, Bl_{turb} from eq. (9)	k_{trans}, W/mK, from eq. (10)	k_{turb}, W/mK, from eq. (11)	δ_{LBL}- average thickness LBL 10^{-3}, m, from eq. (12)	h_{new}, W/m²K, from eq. (15/16)	h_{new} W/m²K, from eq.(25)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
243	1066	3.45	0.397	160	54.41	130 (L)	1390	49.3 49.3	359.4	0.00	172.7	0.00	3.20	0.00	10.5	305	**304.8**
253	1062	3.49	0.396	74.3	52.11	278 (L)	654.8	23.0 23.0	333.9	0.00	84.2	0.00	3.08	0.00	10.5	293	**293.3**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
263	1058	3.52	0.3955	31.74	49.81	649 (L)	282.5	9.86 9.86	307.4	0.00	37.8	0.00	2.96	0.00	10.5	282	**281.9**
273	1054	3.56	0.3950	18.97	47.58	1082 (L)	170.9	5.92 5.92	291.9	0.00	23.8	0.00	2.84	0.00	10.5	271	**270.5**
283	**1049**	**3.59**	**0.3945**	**8.156**	**46.47**	**2504 (Tr)**	**74.2**	**4.47** **4.85**	**1096** **748**	**0.0015** **0.0014**	**10.5**	**0.41** **1.07**	**2.78**	**1.14** **2.97**	**5.07**	**1097**	**1046**
293	1044	3.62	0.3940	6.264	45.36	3245 (Tr)	57.5	4.19 4.44	1240 976	0.0021 0.0013	8.31	1.01 1.38	2.73	2.39 3.78	5.22 4.41	1238 1046	**1200**
303	1033	3.66	0.3935	4.621	43.15	4352 (Tr)	43.0	3.89 4.03	1423 1250	0.0031 0.0025	6.48	1.51 1.86	2.61	3.95 4.86	4.18 3.67	1422 1249	**1382**
313	1030	3.69	0.3930	2.978	40.96	6733 (Tr)	28.0	3.49 3.53	1749 1706	0.0053 0.0051	4.42	2.60 2.69	2.49	6.47 6.69	2.92 2.85	1747 1705	**1792**
323	**1024**	**3.73**	**0.3925**	**2.301**	**38.76**	**8663 (Tr)**	**21.9**	**3.28** **3.28**	**1972** **2001**	**0.0071** **0.0072**	**3.63**	**3.43** **3.37**	**2.37**	**8.14** **7.99**	**2.37** **2.40**	**1975** **2000**	**2031**
333	1015	3.76	0.3920	1.624	36.57	12167 (T)	15.6	3.01 2.98	2367 2434	0.0107 0.0111	2.72	4.70 4.54	2.24	10.5 10.2	1.84 1.89	2370 2435	**2496**
343	1007	3.77	0.3915	1.362	34.36	14393 (T)	13.0	2.89 2.85	2503 2652	0.0128 0.0137	2.42	5.32 5.56	2.10	11.8 11.0	1.66	2530	**2624**
353	999	3.82	0.3910	1.10	32.14	17679 (T)	10.8	2.74 2.70	2714 2965	0.0155 0.0172	2.12	6.82 6.16	1.99	13.6 12.3	1.34 1.47	2707 2970	**2941**
363	992	3.85	0.3905	0.954	29.98	20242 (T)	9.4	2.65 2.61	2854 3177	0.0177 0.0200	1.97	7.96 7.05	1.86	14.8 13.1	1.17 1.30	2861 3179	**2976**
373	984	3.89	0.390	0.807	27.74	23737 (T)	8.0	2.55 2.50	3026 3428	0.0207 0.0238	1.81	9.40 8.18	1.73	16.3 14.2	1.01 1.14	3035 3426	**3038**

* The values in bold font indicate the boundaries of the *L–Tr* and *Tr–T* regime transitions. The critical values of the numbers *Bl* and *Bl_turb* are highlighted on a gray background

$$Nu = \frac{(0.125 \cdot f)(\text{Re} - 1000) \cdot \text{Pr}}{1 + 12.7 \cdot (0.125 \cdot f)^{0.5} \cdot (\text{Pr}^{0,66} - 1)} \quad (T, Tr)$$

$$f_1 = 0.316 / \text{Re}^{0.25} \qquad (3)$$
$$Nu = 0.17 \cdot \text{Re}^{0.33} \cdot Pr^{0.43} \ L \ \text{mode} \qquad (4)$$
$$Nu = 0.008 \cdot \text{Re}^{0.9} \cdot Pr^{0.43} \ Tr \ \text{mode} \qquad (5)$$
$$Nu = 0.021 \cdot \text{Re}^{0.8} \cdot Pr^{0.43} \ T \ \text{mode} \qquad (6)$$

The following columns 11 (Tables 1–3) presents the dimensionless coefficient of turbulence of the coolant flow a, which was calculated from the Eq. (7):[7]

$$a \approx \frac{1}{\sqrt{2\,\text{Re}}} \left(\frac{r \cdot h_{classic}}{2 \cdot \mu \cdot C_P} - \frac{1}{Bl} \right); \qquad (7)$$

where h is the heat transfer coefficient, which was calculated according to classical empirical equations for Tr and T regimes (3), (4), (5), (6) a is the flow turbulence coefficient, the experimental value of which is found in the literature only for the air medium and is $a = (0.05–0.08)$.[6] Formula (7) for the theoretical derivation of the turbulence coefficient a is presented in our paper.[7]

It should be noted that the ratio (7) includes the heat transfer coefficient, which was previously calculated on the basis of classical empirical equations (3)–(6), which were compiled on the basis of powerful experimental material of many independent authors. Therefore, it should be assumed that the ratio (7), that is, the theoretically calculated turbulence coefficient a, also carries a powerful experimental load.

The following columns 12, 13 (Tables 1–3) present the numbers Bl (Bilonoga number) and Bl_{turb} (Bilonoga turbulent number) (Formulas (8), (9) respectively).

Detailed information on the derivation of these numbers by the dimensional analysis method is provided in works,[8, 9] respectively, and with their application to nanofluid coolants in our previous works.[2, 3]

$$Bl = \frac{\mu \cdot \sqrt{C_P \cdot 1K}}{\sigma \cdot \cos\theta_{trans}} = \left[\frac{Pa \cdot s}{Pa \cdot s} = \frac{m/s}{m/s} \right] \qquad (8)$$

where Bl is a dimensionless number; μ is a coefficient of dynamic viscosity of the coolant, kg/m·s; $(\cos\Theta_{trans} \approx 1)$ is a cosine of the angle (Surface hydrophilicity); C_P is a specific heat capacity of the coolant, J/kg·K; σ is a coefficient of surface tension of the coolant, N/m;

$$Bl_{turb.} = \frac{\mu_{turb.} \cdot \sqrt{C_P \cdot 1K}}{\sigma \cdot \cos\theta_{trans}} = \left[\frac{Pa \cdot s}{Pa \cdot s} = \frac{m/s}{m/s} \right] \qquad (9)$$

where Bl_{turb} is a dimensionless turbulent number; μ_{turb} is a coefficient of turbulent viscosity of the coolant, kg/m·s.

In the following columns 14, 15, for the specified simulated flows of coolants (Tables 1–3), the transient k_{trans} and turbulent k_{turb} of thermal conductivity (Fig. 2) are calculated according to the corresponding equations (10) and (11).[8] Detailed information about these quantities is presented in our previous works.[8, 9]

$$k_{trans.} = \sigma \cdot \sqrt{C_P \cdot 1K} \qquad (10)$$

where k_{trans} is an average transient thermal conductivity in laminar boundary layer (LBL).

$$k_{turb} = k_{trans} \cdot Bl_{turb} = \mu \cdot a \cdot \sqrt{2\,\text{Re}} \cdot C_P \ W/m \cdot K \qquad (11)$$

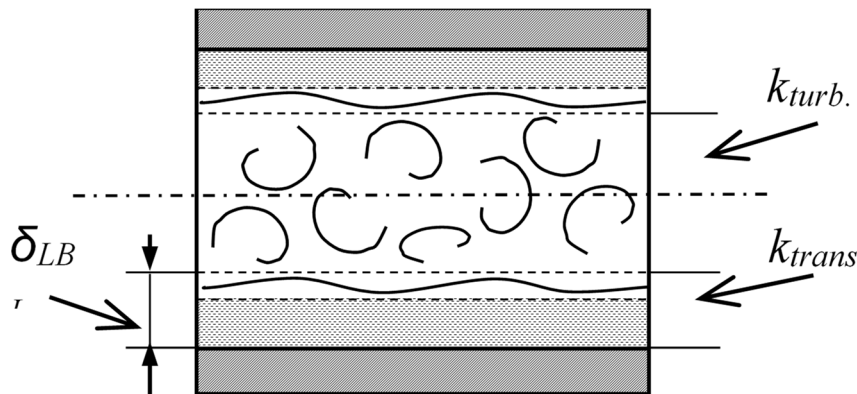where a is the turbulisation coefficient, calculated in columns 11 by formula (7).[7]

Fig. 2. Scheme of the movement of the coolant in *Tr* or *T* regimes with the formation of a LBL

The next columns 16 (Tables 1–3) presents computer calculations of the average thickness of the LBL, which was calculated according to the formula (12), derived by us in the paper,[10] and found its further application in the article.[7]

$$\delta_{LBL} = \frac{r}{Bl_{turb} + 1} \qquad (12)$$

where δ_{LBL} is the average thickness of LBL, m, Bl_{turb} is a dimensionless turbulent number (Bilonoga turbulent number), r is the radius of the pipe space, m.

The following columns 17 (Tables 1–3) present the computer calculation of the heat transfer coefficient *h*, calculated according to the formula (13)[7, 10] or (15):

$$h_{NEW} \approx \left(\frac{\delta_{LBL}}{k_{trans}} + \frac{(r - \delta_{LBL})}{k_{turb}} \right)^{-1} \quad (13)$$

Formula (13) actually represents the sum of the coefficients of thermal conductivity through the LBL and the turbulent part of the coolant flow. Based on the conclusion of formula (13), the thermal resistance of LBL is equal to the thermal resistance of the turbulent part of the coolant flow. That is, with an increase in the flow rate, the average thickness of the LBL decreases, its thermal resistance becomes smaller, and the thermal resistance of the turbulent part also decreases proportionally due to more intense turbulence (14):[7, 10]

$$\frac{k_{trans}}{\delta_{LBL}} = \frac{k_{turb}}{r - \delta_{LBL}} = \frac{k_{trans} \cdot Bl_{turb}}{r - \delta_{LBL}} \quad (14)$$

Based on formulas (12) and (13), the total thermal resistance of the system, or the total heat transfer coefficient for the flow with *Tr* or *T* regime can be calculated by the simple relation (15):

$$h_{NEW\,Tr,T} = 2 \frac{k_{trans}}{\delta_{LBL}} \quad (15)$$

In the *L* mode of the heat carrier movement, formula (15) looks even simpler, since there is no turbulence and the layer through which the amount of heat must pass from the conventional axis of the center of the flow is equal to the radius, and the ratio (15) turns into (16):

$$h_{NEW\,L} = \frac{k_{trans}}{r} \quad (16)$$

Between the values of the turbulent thermal conductivity k_{trurb} and the transient thermal conductivity in the LBL k_{trans} there is an interdependence Bl_{trurb} (Tables 1–3) (17):[8, 2, 3]

$$Bl_{turb} = k_{turb} / k_{trans} \quad (17)$$

The columns 7–17 are obtained through a simulated computer experiment and are also made on the basis of well-known classical numerical power and other equations, which carry a powerful array of experimental data from many independent authors. At the same time, our analytical calculation of heat transfer coefficients includes the previously derived similarity numbers (columns 12, 13, Tables 1–3) *Bl* (Bilonoga number) and Bl_{turb} (Bilonoga number turbulent, formulas (8), (9), respectively).

For computer modeling, the New Microsoft Excel Worksheet program was used, since it is convenient to calculate all the hydromechanical characteristics presented in Tables 1–3, having previously entered formulas for

their calculation into the program. In addition, to enable visual observation of changes and patterns, appropriate graphs were constructed in the same program.

The analysis of the numerical data of the computer simulation is presented below. It should be considered that the results obtained by us in columns 7–17 in Tables 1–3 also carry powerful experimental material, since they were obtained by using classical empirical equations and verified by many experiments over many years.

3. Results and Discussion

It is characteristic of our simulation that for all aqueous solutions of propylene glycol in the simulated flow of the pipe space of the heliosystem (solar heat exchanger with a diameter $D = 0.021$ m and velocity $V = 0.93$ m/s in the temperature range of 243–373 K, this flow has two mode transitions from *L* to *Tr* and further – to *T*, which can be clearly seen by evaluating the numbers Reynolds (marked in bold, Tables 1, 2, 3). In Figs. 3–5 these limits are shown.

At the beginning of the simulation, by analogy with the very precise and reliable experimental studies of the authors of the paper,[4] we made a computer calculation of the friction coefficients *f* (Tables 1–3, column 9) according to known empirical ratios (1 and 2).

For all presented solutions in Tables 1–3 there is a characteristic jump $L \rightarrow Tr$ of the regime transition. There is a complete analogy between our computer simulation and the experiments of work,[4] where at the conventional point $L \rightarrow Tr$ of the mode transition, a sharp jump in the friction coefficient is observed in the range of Reynolds numbers $2300 < Re < 3000$.

At the limit of the $L \rightarrow Tr$ regime transition in the range of Reynolds numbers close to $3000 \geq Re \geq 2300$, the following dependencies can be seen (Tables 1–3):

1. When using the Gnelinsky formula (3) and empirical power classical numerical Eqs. (4)–(6), the number Bl_{turb} according to calculations approaches 1, *i. e.*, $Bl_{turb} \approx 1$.

2. Based on relation (17), in this range of Reynolds numbers, the turbulent thermal conductivity k_{trurb} is close to the average transient thermal conductivity in the LBL k_{trans}, *i. e.*, $k_{trurb} \approx k_{trans}$.

3. Based on formula (12), the average thickness of LBL will be equal to: $\delta_{LBL} \approx r/1+1$. At the same time, it is obvious that at such small values of the Reynolds number, that is, at the beginning of the *Tr* regime, it occupies approximately half the radius of the "live section" of the flow.

4. For all tables (Tables 1–3) at the boundary of the $L{\rightarrow}Tr$ regime transition, the molecular number Bl at the turbulent number $Bl_{turb} \approx 1$ is approximately equal to 10, *i. e.*, $Bl_{critical} \approx 10.47$ (Marked in bold, Tab. 1–3). Such a result led us to believe that the critical Bilonoga number $Bl_{critical}$ at the $L{\rightarrow}Tr$ regime transition, when the first vortices of free turbulence appear in the flow, is numerically equal to the so-called modified Reynolds number on the surface of the LBL, which can be found in literary sources using formula (18) and it varies in the range $N = (10.47–11.5)$.[11]

$$N = \frac{V_X \cdot \delta_{LBL} \cdot \rho}{\mu} \qquad (18)$$

where V_X is the average flow velocity in the LBL layer, m; δ_{LBL} is the average thickness of LBL, m; ρ is the heat carrier density, kg/m³; μ is the coefficient of dynamic viscosity of the coolant, Pa·s.

The so-called turbulence constant N, which is today called the friction Reynolds number, and which, based on Prandtl's turbulence theory,[12, 13] was derived by taking the logarithm of the mixing velocity.

However, in our works, as well as in the works of G. Reichardt,[14–16] no theories of turbulence were applied. However, we came across a value identical in numerical value after previously calculating the molecular number of Bilinoga Bl,[9] which, as it turned out in this work, at the $L{\rightarrow}Tr$ regime transition at the turbulent number $Bl_{turb} \approx 1$, is equal to: $Bl_{critical} \approx 10.47$. At the same time, we used only well-known thermophysical characteristics, which are included in the number of Bl (formula 8), and this pattern is observed for all three coolants (Tables 1–3). It is very important that, using the concepts of molecular physics and surface physics, we come to an identical result from the point of view of fluid mechanics, where tangential stresses, mixing length, and logarithmization of velocity in LBL according to Prandtl's theory are considered.[12, 13] In our opinion, this shows that our approach is constructive and correlates well with classical fluid mechanics results.

5. From all three tables (Tables 1–3) it can be seen that near the $L{\rightarrow}Tr$ regime transition the values for all values calculated according to Gnelinsky's equation (3) differ significantly from the values calculated according to relation (5). In our previous study, we proved that it is not possible to use equality (3) in the vicinity of this transition, as it gives a false result (see Fig. 2).[7] Therefore, at points close to the $L{\rightarrow}Tr$ transition, we take into account only those values obtained from Eq. (5).

At the boundary of the $Tr{\rightarrow}T$ regime transition in the range of Reynolds numbers close to $10000 \geq Re \geq 8000$, the following dependencies are observed (Tables 1–3):

1. Using Gnelinsky's formula (3) and empirical power classical numerical Eqs. (4)–(6), the turbulent number Bl_{turb} is calculated to be close to the molecular number Bl, *i. e.*, $Bl_{turb} \approx Bl$.

2. The relation between turbulent and transient thermal conductivity (17) is completely true, *i. e.*, $k_{trurb} \approx k_{trans.} \cdot Bl_{trurb}$

3. The average thickness of LBL, based on formula (12), is equal to: $\delta_{LBL} \approx r/Bl+1$.

Interdependencies between molecular Bl and turbulent Bl_{turb} numbers for all three propylene glycol solutions are presented in (Figs. 3–5).
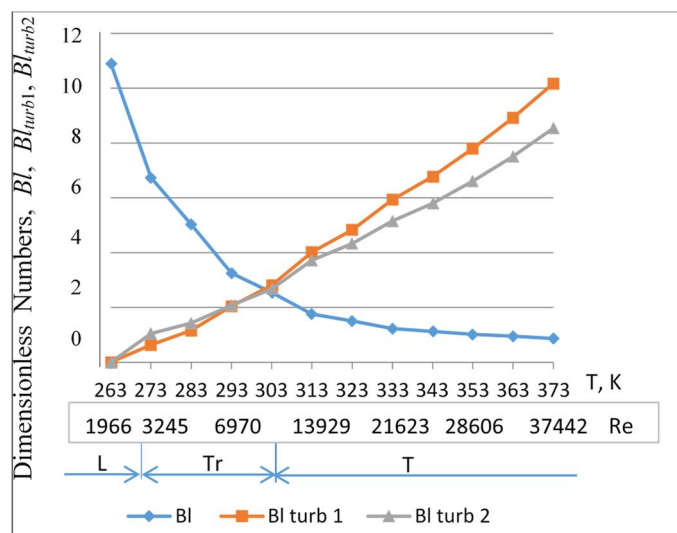

Fig. 3. Dependence of molecular number Bl and turbulent numbers Bl_{turb}.
for an aqueous solution of propylene glycol (25 %), moving in a pipe space with $D = 0.021$ m
of a heliosystem and $V = 0.93$ m/s from the Reynolds number in the temperature range of 263–373 K (Table 1)
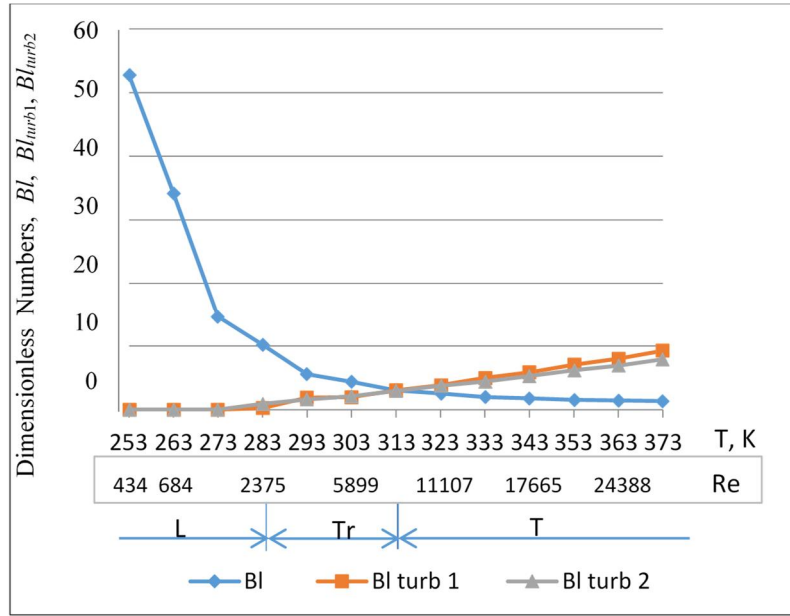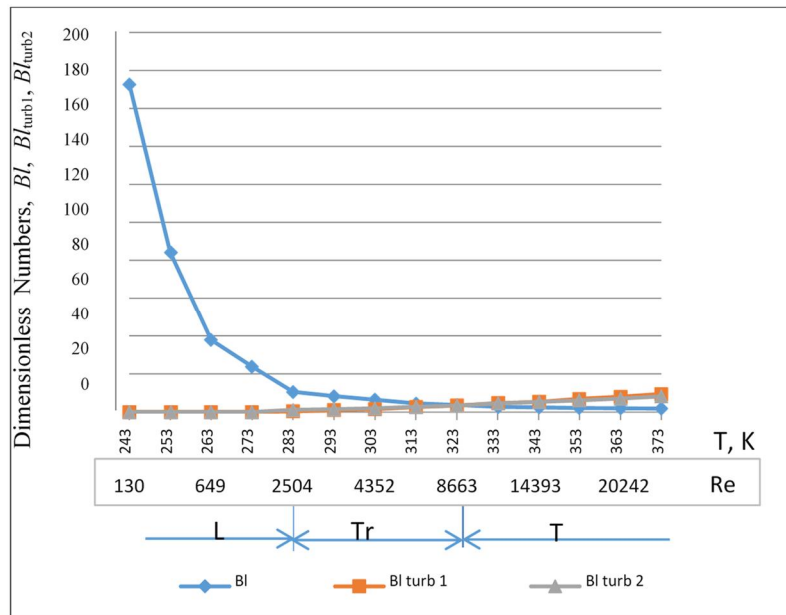
Fig. 4. Dependence of molecular number Bl and turbulent numbers Bl_{turb}.
for an aqueous solution of propylene glycol (37 %), moving in a pipe space with $D = 0.021$ m
of a heliosystem and $V = 0.93$ m/s from the Reynolds number in the temperature range of 263–373 K (Table 2)

Fig. 5. Dependence of molecular number Bl and turbulent numbers Bl_{turb}.
for an aqueous solution of propylene glycol (45 %) moving in a pipe space with $D = 0.021$ m
of a heliosystem and $V = 0.93$ m/s from the Reynolds number in the temperature range of 263–373 K (Table 3)

It can be seen from Figs. 3, 4, and 5 how the graphs intersect at the conditional points $Tr{\rightarrow}T$ of the mode transitions of all coolants for Reynolds numbers in the small range $8000 < Re < 9000$. This suggests that our method can also be used to quickly search for the critical transition points of the $L{\rightarrow}Tr$ and $Tr{\rightarrow}T$ regimes, and that our results correlate well with the classical ones.

Based on the formula (18) and the values of the modified Reynolds number $N = (10.47{-}11.5),$[11] the average flow velocity in the LBL (19):

$$V_X = \frac{N \cdot \mu}{\delta_{LBL} \cdot \rho} = \frac{(10,5 - 11,5) \cdot \mu}{\delta_{LBL} \cdot \rho} \qquad (19)$$

where δ_{LBL} is the average thickness of the LBL layer, m, from relation (12); $N = (10.47–11.5)$ is the modified Reynolds number on the surface of the LBL.[11]

And, finally, the main regularity: when the first signs of turbulence appear, that is, the number $Bl \approx 10.47$, and the number $Bl_{turb} \approx 1$ and further, as the temperature increases, the number Bl gradually decreases, and the number Bl_{turb} increases smoothly (Figs. 1–3) in the temperature range where Tr and T regimes are present (Tables 1–3), dependence (20) arises:

$$Bl \cdot Bl_{turb} \approx N \approx 10.47–11.5. \qquad (20)$$

Numerous experiments show that the heat flux density at each point is directly proportional to the temperature gradient at that point, which is reflected by the classical Fourier heat conduction equation (21):[17]

$$jQ = k\frac{dT}{dx} \qquad (21)$$

where jQ is the heat flow density, dT/dx is the temperature gradient, k is the thermal conductivity of the medium.

Obviously, the temperature of the medium at a certain point is directly proportional to the thermal conductivity of the medium at that point. Then the ratio of temperatures in the center of the flow and in the LBL is directly proportional to the ratio of turbulent thermal conductivities in the center of the flow and transient thermal conductivities in the LBL, and this statement is analogous to the equation of the Reichardt distribution law presented in known works[14–16] (22) (Fig. 6):

$$Bl_{turb} = \frac{k_{turb.}}{k_{trans/.}} = \frac{T_{MAX}}{T_{MIN}} = \left(\frac{V}{V_X}\right)^{0,5} \qquad (22)$$

where $k_{turb.}, k_{trans.}$ are the maximum and minimum thermal conductivity in the center of the flow axis and in the LBL, W/m·K; T_{max}, T_{min} are the maximum and minimum temperature in the "live section" of the flow of the coolant, V, V_X are the average flow velocity in the center of the flow and in the LBL, respectively.

Such a distribution of velocities and temperatures along the "live cross-section" of the flow in conditions of free turbulence is also indicated by well-known experiments in the theoretical work of G. I.Taylor, conducted by A. Fage and V. M. Falkner.[18]
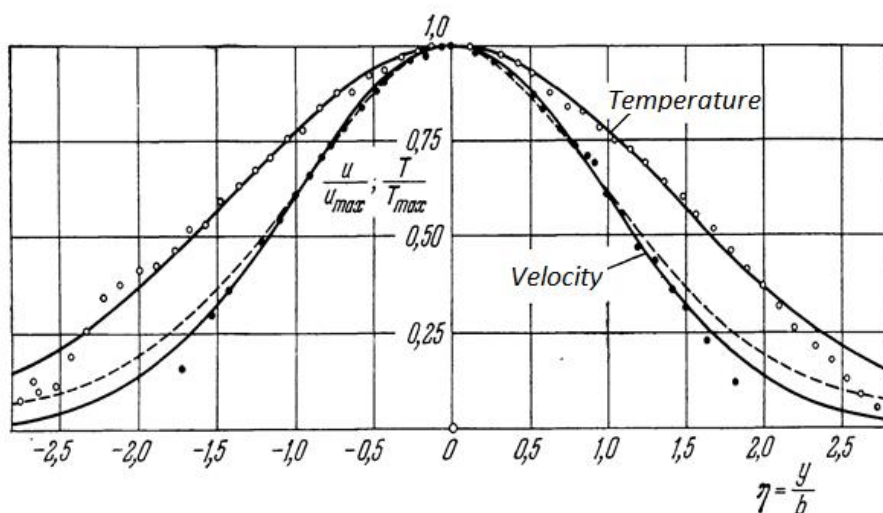

Fig. 6. Distribution of temperatures and velocities in a flow with free turbulence according to H. Reichardt[14–16]

Based on the above, we write the following system of three equations (23):

$$
\begin{cases}
1) & Bl_{turb} = \dfrac{1}{Bl}\left(\dfrac{V}{V_X}\right)^{0,5} & \text{from (22)} \\
\\
2) & V_X = \dfrac{N \cdot \mu}{\delta_{LBL} \cdot \rho} = \dfrac{(10,5-11,5)\cdot \mu}{\delta_{LBL} \cdot \rho} & \text{from (19)} \\
\\
3) & \delta_{LBL} = \dfrac{r}{Bl_{turb}+1}; & \text{from (12)}
\end{cases}
\qquad (23)
$$

We solve this system with respect to the number of $Bl_{turb.}$, according to Viett's formula, obtaining a cubic numerical power Eq. (24):

$$2\cdot N\cdot Bl^2\cdot Bl^3_{turb.} + 2\cdot N\cdot Bl^2\cdot Bl^2_{turb.} - Re = 0; \text{ or}$$
$$2\cdot(10.47–11.5)Bl^2\cdot Bl^3_{turb.} + 2\cdot(10.47–11.5)$$
$$Bl^2\cdot Bl^2_{turb.} - - Re = 0. \qquad (24)$$

Since the change in the product $Bl\cdot Bl_{turb}$ occurs in proportion to the change in the Re number under the influence of temperature, which in turn affects all the thermal-hydrophysical parameters of heat carriers, then in the cubic equation (24) we introduced an additional term

through computer simulation, where Bl_{turb} is a power. Then equation (24) turns into (25):

$$2 \cdot (10.47 - 11.5) \cdot Bl^2 \cdot Bl^3_{turb} + 2 \cdot (10.47 - 11.5)$$
$$Bl^2 \cdot Bl^2_{turb} \pm Re_{kr} \, Bl_{turb} - Re = 0, \qquad (25)$$

where Re is the Reynolds number; N is the modified Reynolds number in the LBL; Bl is the Bilonoga molecular number (formula (8)); Bl_{turb} is the Bilonoga turbulent number (formula (9)); Re_{kr} is the critical Reynolds number equal to 2300.

Solving this cubic equation (online), we get one real root – this is the Bl_{turb} number. The remaining two roots are complex numbers that we do not take into account. Next, using formula (12), we get the average thickness $\delta_{LBL} = \dfrac{r}{Bl_{turb} + 1}$; and finally, we determine the heat transfer coefficient according to the formula (15)

$$h_{NEW \, Tr,T} = 2 \frac{k_{trans}}{\delta_{LBL}}.$$

Based on the above, we presented graphs of the distribution of turbulent thermal conductivities along the "live cross-section" of the tube space of the heliosystem, as well as the distribution of velocities for a 37 % aqueous solution of propylene glycol moving in the T regime with

$V = 0.93$ m/s and $D = 0.021$ m at a temperature in the core of the flow of 343 K (Fig. 7).

For this, we determined the distribution of velocities along the "live section" of the pipe space of the heliosystem, taking a step of $\Delta = 1 \cdot 10^{-3}$ m along the radius r of the pipe from the axis to the left and right to the walls (Fig. 7). In the center of the axis T of the core of the flow, the velocity is maximum ($V = 0.93$ m/s) and it gradually decreases, approaching zero on the inner surface of the wall (left scale, Fig. 7).

Next, according to the velocity values, we calculated the distribution of Reynolds numbers, since all other parameters (density, pipe diameter, and dynamic viscosity coefficient) remained constant for a particular 37 % aqueous solution of propylene glycol at a temperature of 343 K. Next, we inserted these values of the Reynolds numbers at the points from the core axis of the heliosystem tube flow to the walls into the cubic equation (24) we derived and found (online) the only real value of the roots, *i. e.*, the number of Bilonoga turbulent Bl_{turb} for each velocity vector, respectively. Having received ten values of Bl_{turb} and based on formula (17), ten values of turbulent thermal conductivities k_{turb} were calculated, respectively, from the flow axis to the left and right to the walls (right scale, Fig. 7).
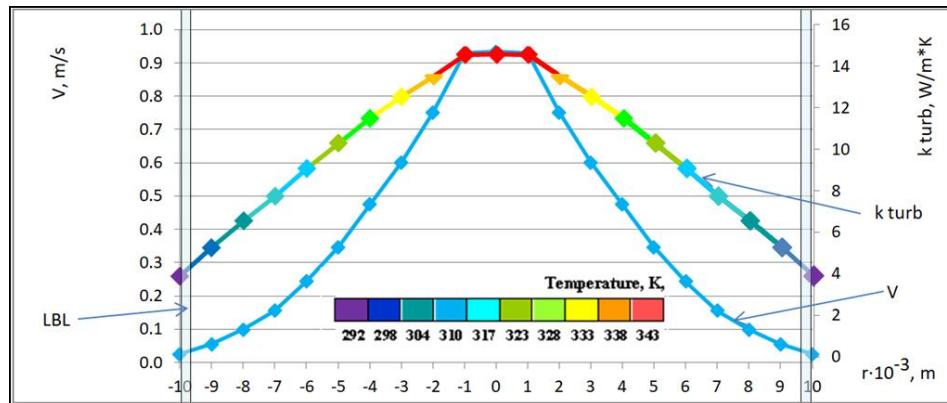

Fig. 7. Distribution of turbulent thermal conductivities k_{turb} (W/m·K) (as well as temperatures, T, K) and velocities V (m/s) in a flow with free turbulence for an aqueous solution of 37 % propylene glycol at an axial velocity in the center of the flow core $V = 0.93$ m/s at a temperature of 343 K in the pipe space of the heliosystem with $D = 0.021$ m

According to the obtained points both for the values of turbulent thermal conductivities kturb and for velocity vectors V plotted the distribution of velocities and turbulent heat conductivities k_{turb} to the left and right of the center of the coolant flow axis (Fig. 7). According to the classical Fourier equation of thermal conductivity (21),[17] the distribution of temperatures along the radius of the "live section" of the flow is directly proportional to the distribution of thermal conductivities (turbulent). We placed the scale of the corresponding temperatures in degrees Kelvin, corresponding to the values of turbulent

thermal conductivities, below the graphs. (Color under the graphs) (Fig. 7). If we consider a different coolant and a different temperature in the center of the core, then the layer-by-layer nature of changes in velocities and turbulent thermal conductivities of this flow from the center of the core to the walls will be similar.

It can be seen from (Fig. 7) that the graphs are similar to the experimental results obtained by H. Reichardt (Fig. 6), and A. Fage and V. M. Falkner, and presented in works,[14–16, 18] as well as classic works of several authors, for example, B. A. Bakhmetev and

L. G. Loitsyanskyi. The distribution of velocities is subject to a square parabola, and the distribution of turbulent thermal conductivities according to the numerical cubic equation (24), (25), and, accordingly, of temperatures is cubic (Fig. 7). This indicates the maximum correlation of the results obtained by us in this and previous works,[2,3,7-11] based on the laws and characteristics of fluid physics and surface physics with classical works obtained by many authors from the standpoint of fluid mechanics and compactly collected in monographs of H. Schlichning[19] (see pages 706–707,752–753,741,747,749,754), including the well-known works of H. Reichardt, G. Taylor and many others. The advantages of the analytically obtained equation (24), (25) in comparison with the classical numerical empirical equations (3)–(6), in particular for the calculation of heliosystems with nanofluid coolants, will be considered by us in the following works with the possibility of applying this approach in the calculations of solid-gas systems, for example in conditions of filtration drying.[20, 21]

We presented the analytical cubic numerical equation (24), (25) for the general solution. However, in heliosystems, the *Tr* regime is dominant. Therefore, for a quick assessment and calculation of heat transfer coefficients for *Tr* mode, you can use equation (20) and, having obtained the value of the Bl_{turb} number according to formulas (12), (15), find the value of the heat transfer coefficient. For mode *L*, equality (16) should be used. Column 18 shows the results of computer modeling for determining the heat transfer coefficient according to the new analytical cubic numerical equation (24), (25) that we obtained, the numerical data of which correlate well with the classical ones given in columns 17 (Tables 1–3). The ratio between the new similarity numbers Bl and Bl_{turb} correlate well with the results of the modern works of Italian specialists in the field of fluid mechanics M. Quadrio and P. Ricco, in particular, are consistent with their well-known work,[22] where the dominance of the *L* part of the flow over the turbulent part is theoretically proven, that is, the *L* part of the flow completely controls the *T* zone, which is clearly seen from the interdependencies we derived (13–17) in previous works.[2,3,7–11]

4. Conclusions

1. Computer simulation of the movement of three coolants (25 %, 37 %, 45 %) aqueous solutions of propylene glycol in the pipe space of the solar system with a diameter of $D = 0.021$ m and a constant velocity $V = 0.93$ m/s was carried out.

2. Two regime transitions $(L \rightarrow Tr)$ and $(Tr \rightarrow T)$ occur in the three coolants in the temperature range of 243–373 K.

3. The results obtained by us are similar to the experimental results of H. Reichardt, as well as A. Fage and V. M. Falkner, as well as classic works of several other authors. The distribution of velocities in the general flow of the heat carrier obeys a square parabola, and the distribution of turbulent thermal conductivities and, accordingly, temperatures, a cubic parabola. This testifies to the maximum correlation of the results obtained by us in this and previous works, based on the laws and features of fluid physics and surface physics, with classical works obtained by numerous authors from the standpoint of fluid mechanics.

4. Friction coefficients were calculated in the specified temperature range. During the mode transition $(L \rightarrow Tr)$, jumps are formed in the same way as for the classical experimental studies.

5. The critical points of mode transitions $(L \rightarrow Tr)$ and $(Tr \rightarrow T)$ are fixed not only by the critical values of the Reynolds numbers we are used, but also by the critical values of the Bl and Bl_{turb} numbers (Figs. 3–5, where the graphs intersect), *i. e.*, $Bl \approx Bl_{turb}$.

6. Based on the system of equations (23), a new cubic numerical criterion equation (25) is constructed, which is solved analytically with respect to the number Bl_{turb}. and has one real root. Further, by solving the system of equations (23), it is possible to analytically obtain the values of the heat transfer coefficients.

7. Having obtained the values of heat transfer coefficients by analytical method (bypassing laborious and expensive empirical studies), it is possible to design and build solar systems of appropriate performance, as well as select effective coolants, in particular with nanofluid additives. This approach will be the subject of further research.

8. The correlation between the new similarity numbers Bl and Bl_{turb} correlates well with the results of modern works by Italian researchers in the field of fluid mechanics M. Quadrio and P. Ricco, where the dominance of the *L* part of the flow over *T* was theoretically proven, *i.e.*, the *L* zone completely controls the *T* part, which is clearly visible from the interdependencies 13–17 derived by us in the previous studies.

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ТЕПЛОФІЗИЧНИЙ АНАЛІЗ (25 %, 37 %, 45 %) ВОДНИХ РОЗЧИНІВ ПРОПІЛЕНГЛІКОЛЮ ДЛЯ ГЕЛІОСИСТЕМ ТА ЇХНІЙ АНАЛІТИЧНИЙ РОЗРАХУНОК

Анотація. *У роботі проаналізовано теплофізичні властивості водних розчинів (25 %, 37 %, 45 %) пропіленгліколю для геліосистем як теплоносіїв. Здійснено комп'ютерне моделювання руху цих теплоносіїв у трубному просторі геліосистеми зі сталою швидкістю V = 0,93 м/с за допомогою класичних числових емпіричних рівнянь для знаходження теплофізичних і гідродинамічних характеристик водних розчинів вказаних гліколів у інтервалі температур 243–373 К. Розподіл векторів швидкостей у "живому перерізі" трубного простору геліосистеми підпорядкований квадратній параболі, а розподіл турбулентних теплопровідностей і, відповідно, температур – кубічній. Аналітично отримано кубічне числове рівняння, дійсним коренем якого є безрозмірне число Bl$_{turb}$, що використовують для знаходження коефіцієнтів тепловіддачі теплоносіїв за будь-якої температури та швидкості руху в турбулентному та перехідному режимах. Отримано співвідношення для знаходження чисел Bl$_{turb}$ для перехідних режимів руху, які здебільшого реалізуються в геліосистемах. Графічно відображено розподіл турбулентних теплопровідностей k$_{turb}$ (Вт/м·К) (а також температур, Т К) і швидкостей V (м/с) у потоці з вільною турбулентністю для водного розчину 37 % пропіленгліколю за швидкості в центрі ядра потоку V = 0,93 м/с та температури 343 К у трубному просторі геліосистеми діаметром D = 0,021 м.*

Ключові слова: перехідна, турбулентна в'язкість і теплопровідність, середня товщина ламінарного примежового шару (ЛПШ), коефіцієнт поверхневого натягу теплоносія, аналітичне кубічне числове рівняння.